**Total marks (120)**
**Attempt Questions 1 – 10**
**All questions are of equal value**

Answer each question in a SEPARATE writing booklet.

**QUESTION 1** (12 MARKS) Use a SEPARATE writing booklet **Marks**

(a) $f(x) = 2x^3 - 3x$. Evaluate $f(-3)$. **1**

(b) Simplify $2 \times |-5| - |-12|$. **1**

(c) Simplify $\dfrac{4x^2 - 9}{4x + 6}$. **2**

(d) Rationalise the denominator and simplify $\dfrac{\sqrt{3} - 1}{\sqrt{3} + 1}$. **2**

(e) Find the values of $\dfrac{\tan x}{x + 1}$ when $x = 2.3$ radians, giving your answer correct to two decimal places. **2**

(f) The length of a rectangle is increased by 5% and the width is decreased by 2%. Find the percentage increase in its area. **2**

(g) Solve the inequality $2x^2 + 7x - 4 \geq 0$. **2**

**QUESTION 2** (12 MARKS) Use a SEPARATE writing booklet **Marks**

(a) Differentiate the following functions:

(i) $3x^2 - 2\sqrt{x}$. **2**

(ii) $\dfrac{1}{(3x-2)^2}$. **2**

(b) Find $\int \left(e^{2x} - \sin x\right) dx$. **2**

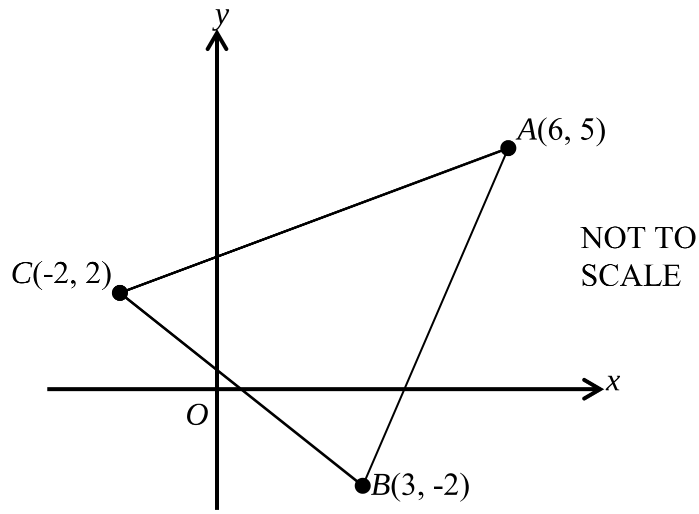
(c) Evaluate $\int_4^5 \frac{1}{x-3}\, dx$ correct to two decimal places. **2**

(d) (i) Differentiate $x^2 \ln x$. **2**

(ii) Hence find $\int x \ln x \, dx$. **2**

**QUESTION 3** (12 MARKS) Use a SEPARATE writing booklet **Marks**

(a) The diagram shows the three points $A(6, 5)$, $B(3, -2)$ and $C(-2, 2)$.



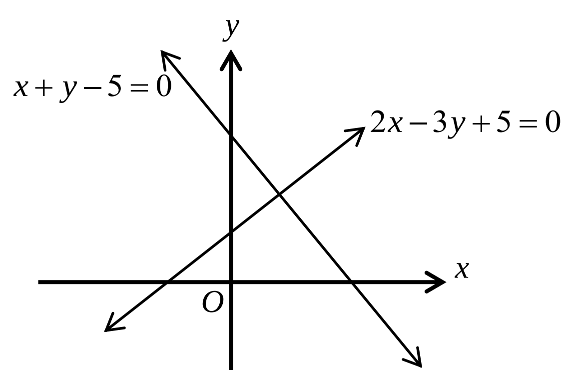

(i) Find the length of $AC$. **1**

(ii) Show that the equation of $AC$ is $3x-8y+22=0$. **2**

(iii) Find the perpendicular distance from $B$ to $AC$. **2**

(iv) Hence find the area of $\Delta ABC$. **2**

(b) The diagram shows the graphs of the lines $x+y-5=0$ and $2x-3y+5=0$.




(i) Find the coordinates of the point of intersection of the two lines. **3**

(ii) Copy the diagram into your writing booklet, and shade the region which satisfies both the inequalities $x+y-5\le 0$ and $2x-3y+5\le 0$. **2**

**QUESTION 4** (12 MARKS) Use a SEPARATE writing booklet **Marks**

( a) Solve the equation $2^{2x} - 7 \times 2^{x} - 8 = 0$. **3**

(b) (i) Write down the discriminant of the expression $kx^2 + 4x + k$. **1**

(ii) For what values of $k$ does the equation $kx^2 + 4x + k = 0$ have no real roots? **2**

(c) The equation of a parabola is $x^2 = -8(y+2)$.

(i) Write down the coordinates of the vertex of the parabola. **1**

(ii) What is the focal length of the parabola? **1**

(iii) Write down the coordinates of the focus of the parabola. **1**

(iv) What is the equation of the directrix of the parabola? **1**

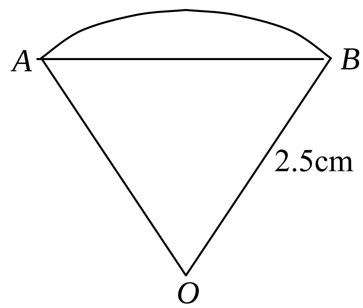
(d) If $\alpha, \beta$ are the roots of the quadratic equation $ax^2 + bx + c = 0$, prove that **2**

$$(\alpha - \beta)^2 = \frac{b^2 - 4ac}{a^2}.$$

**QUESTION 5** (12 MARKS) Use a SEPARATE writing booklet **Marks**

(a) In $\Delta PQR$, $PQ = 12$cm, $QR = 8$cm, and the area of the triangle is 28.8 cm². **3**
Find two possible values for the size of $\angle PQR$ (to the nearest degree).

(b)




NOT TO SCALE

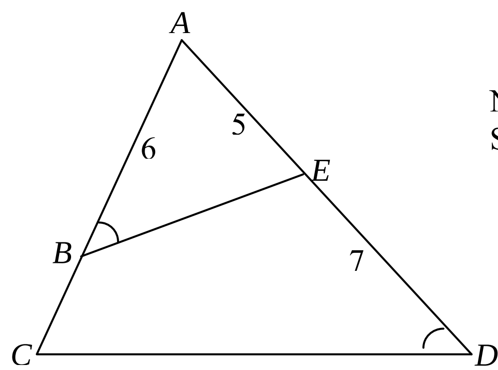
The curve $AB$ is the arc of a circle, centre $O$ and radius 2.5cm.
The length of the arc is 4cm.

(i) Show that $\angle AOB = 1.6$ radians. **1**

(ii) Find the length of the chord $AB$ (to one decimal place). **2**

(iii) Find the area of the minor segment formed by arc $AB$ and the chord $AB$. **2**

(c)




NOT TO SCALE

In the diagram, $\angle ABE = \angle ADC$, $AE = 5$cm, $DE = 7$cm, $AB = 6$cm.

(i) Prove that $\Delta ABE$ is similar to $\Delta ACD$. **2**

(ii) Hence find the length of $BC$. **2**

**QUESTION 6** (12 MARKS) Use a SEPARATE writing booklet **Marks**

(a) In an arithmetic series, the sum of the first 16 terms is 288 and the sixth term is 8. **4**
Find the first three terms of the series.

(b) Find the least positive integer $n$ that satisfies the inequation **3**

$$4 \times (1.2)^n > 560.$$

(c) \$1500 is deposited in an account at the start of each year, the first deposit being in 2006 and the last in 2015.
The account pays interest at 8% p.a., compounded half-yearly.

(i) Find the value of the first deposit at the end of 2015. **2**

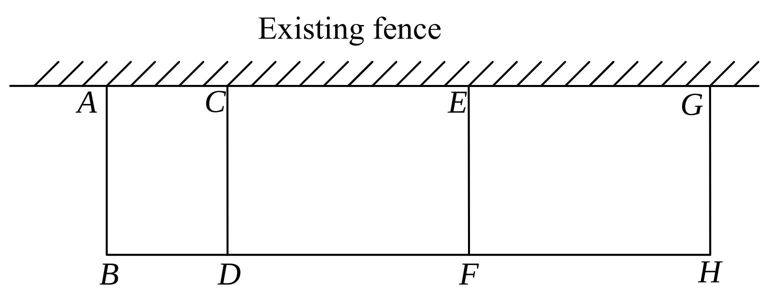
(ii) Find the total value of the account at the end of 2015. **3**

**QUESTION 7** (12 MARKS) Use a SEPARATE writing booklet **Marks**

(a) Find the equation of the normal to the curve $y = 3x^2 - 8x + 2$ at the point (2, -2) on the curve. **3**

(b) Show that the curve $y = x^3 - 12x^2 + 48x + 50$ has only one stationary point , and show that it is a horizontal point of inflexion. **4**

(c)




A rectangular yard is to be constructed using an existing fence as one side, and the yard is to be divided into three rectangular regions, as shown in the diagram.
40 metres of fencing is to be used to construct the five lengths of fencing, that is: *AB, CD, EF, GH*, and *BH*.

(i) If the length of *AB* is $x$ metres, find the length of *BH* as a function of $x$. **1**

(ii) Hence find the dimensions of the yard so that the total area is a maximum. **4**

**QUESTION 8** (12 MARKS) Use a SEPARATE writing booklet **Marks**

(a) (i) State the period of the function $y = 1 + \sin 2x$. **1**

(ii) Hence sketch the graph of $y = 1 + \sin 2x$ for $-\pi \le x \le 2\pi$. **3**

(b) If $\cos\theta = -\frac{2}{3}$ and $\tan\theta > 0$, find the exact value of $\sin\theta$ (without fnding the value of $\theta$). **2**

(c) (i) Sketch the graph of $y = \cos x$ for $0 \le x \le 2\pi$. **1**

(ii) Hence solve the inequation $\cos x \le \frac{\sqrt{3}}{2}$ for $0 \le x \le 2\pi$. **2**

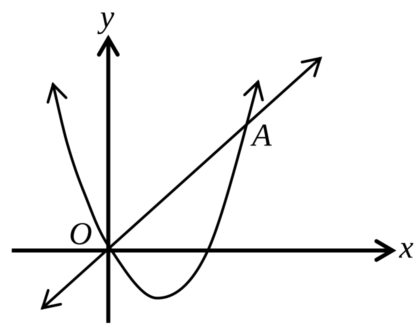
(d) (i) Given the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$, what operation will produce the identity $\tan^2\theta + 1 = \sec^2\theta$? **1**

(ii) Hence find $\int (\tan^2 x)\, dx$. **2**

**QUESTION 9** (12 MARKS) Use a SEPARATE writing booklet **Mark**

(a) The arc of the graph of $y = 4 - x^2$ between (0, 4) and (3, -5), is rotated about the $y$ axis. Find the volume of the solid formed. **3**

(b)




The line $y = 2x$ and the parabola $y = x(x-3)$ meet at $O(0, 0)$ and $A$.

(i) Find the $x$ coordinate of $A$. **1**

(ii) Hence find the area bounded by the line and the parabola. **3**

(c) A function $y = f(x)$ has the following function values:

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 0 | 2 | 4 | 6 | 8 |
| $f(x)$ | 3.2 | 1.4 | 0.6 | 2.2 | 3.6 |

(i) Use Simpson's Rule with five function values to find the approximate value of **2**

$$\int_0^8 f(x)\,dx.$$

(ii) The graph of $y = f(x)$ for $0 \le x \le 8$ is rotated about the $x$-axis. Use Simpson's Rule and five function values to find the approximate volume of the solid formed. **3**

**QUESTION 10** (12 MARKS) Use a SEPARATE writing booklet **Marks**

(a) The number of bacteria $N$ in a colony after $t$ minutes, is given by

$$N = 2000\,e^{0.005t}$$

(i) Find the number of bacteria when $t = 10$. **1**

(ii) Find the rate at which the colony is increasing when $t = 10$. **2**

(iii) Find the time taken for the population to double. **1**

(b) A particle moves along a straight line. Its position $x$ metres from $O$, $t$ seconds after starting, is given by $x = 2t - \log_e(4t+1)$

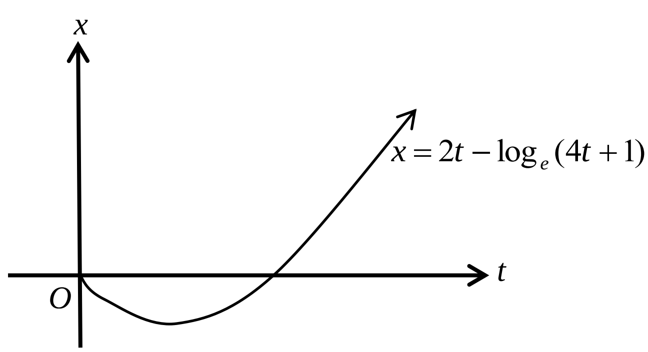
(i) Show that the velocity, $v$ m/sec, and the acceleration, $a$ m/sec², are : **2**

$$v = \frac{8t-2}{4t+1} \quad \text{and} \quad a = \frac{16}{(4t+1)^2}\,.$$

(ii) Find the initial velocity <u>and</u> the initial acceleration. **2**

(iii) Find when the velocity is zero. **1**

(iv)




The diagram shows the displacement-time graph.
On separate number planes, showing relevant features, draw:

($\alpha$) the velocity – time graph **2**

($\beta$) the acceleration – time graph. **1**

**End of paper**

## STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \ a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \ a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \ a \neq 0$$

$$\int \sec ax \, \tan ax \, dx = \frac{1}{a} \sec ax, \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \ a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

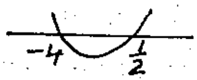
# TRIAL H.S.C.

# EXAMINATION
# 2006

# "MATHEMATICS"

SOLUTIONS AND MARKERS COMMENTS.

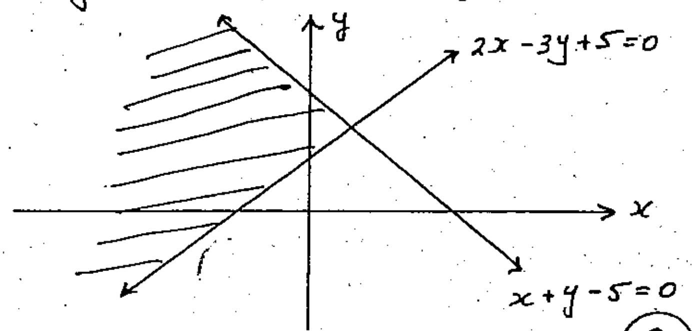
## Mathematics: Question 1

| Suggested Solutions | Marks Awarded | Marker's Comments |
| --- | --- | --- |
| (a) $f(x) = 2x^3 - 3x$ <br> $f(-3) = 2(-3)^3 - 3(-3)$ <br> $= -45$ <br> (1) | 1 | No problems |
| (b) $2 \times \lvert -5 \rvert - \lvert -12 \rvert = 10 - 12$ <br> $= -2$ <br> (1) | 1 | No problems |
| (c) $\dfrac{4x^2-9}{4x+6} = \dfrac{(2x-3)(2x+3)}{2(2x+3)}$ | 1 | Correct factoring |
| $= \dfrac{2x-3}{2}$ <br> (2) | 1 | Correct simplify |
| (d) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1}$ | 1 | Correct first step. |
| $= \dfrac{3-2\sqrt{3}+1}{3-1}$ <br> $= \dfrac{4-2\sqrt{3}}{2}$ | | Many did not simplify FULLY. |
| $= 2-\sqrt{3}$ <br> (2) | 1 | |
| (e) $\dfrac{\tan x}{x+1} = \dfrac{\tan 2.3}{2.3+1}$   NB Radians | 1 | Correct substitution |
| $= -0.34$ (2 d.p.) <br> (2) | 1 | Correct rounding. |
| (f) New area $= (1.05x)(0.98y)$ <br> $= 1.029xy$ | 1 | Correct interpretation |
| where $x, y$ are length and breadth of original rectangle. <br> $\therefore$ Area increases by $2.9\%$ <br> (2) | 1 | Many did not indicate correct increase of $2.9\%$. |
| (g) $2x^2 + 7x - 4 \geq 0$ | 1 | Correct factoring |
| $(2x-1)(x+4) \geq 0$  <br> $x \leq -4, \; x \geq \frac{1}{2}$ <br> (2) | 1 | Correct values AND inequality signs. |

Mathematics: Question 2

| Suggested Solutions | Marks Awarded | Marker's Comments |
|---|---|---|
| (a) (i) $\frac{d}{dx}(3x^2-2\sqrt{x}) = \frac{d}{dx}(3x^2-2x^{\frac{1}{2}})$ $= 6x - x^{-\frac{1}{2}}$ OR $6x - \frac{1}{\sqrt{x}}$ (2) | | |
| (ii) $\frac{d}{dx}\frac{1}{(3x-2)^2} = \frac{d}{dx}(3x-2)^{-2}$ $= -2(3x-2)^{-3}\times 3$ $= \frac{-6}{(3x-2)^3}$ (2) | | Students who used Quotient R. made many more errors. |
| (b) $\int(e^{2x}-\sin x)\,dx = \frac{1}{2}e^{2x}+\cos x + C$ (2) | | Check answer by Diff.... this avoids common error of sign or multiplier of 2. |
| (c) $\int_4^5 \frac{1}{x-3}\,dx = \left[\log_e(x-3)\right]_4^5$ $= \log_e 2 - \log_e 1$ $= \log_e 2$ (2) | | |
| (d) (i) $\frac{d}{dx}x^2\ln x = \ln x\times(2x) + x^2\times\frac{1}{x}$ $= 2x\ln x + x$ (2) | | Use Product R. |
| (ii) $2x\ln x = \frac{d}{dx}x^2\ln x - x$ $x\ln x = \frac{1}{2}\frac{d}{dx}x^2\ln x - \frac{1}{2}x$ $\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$ (2) | | 1 mark for recognising $\int x\ln x dx = \frac{1}{2}x^2\ln x \ldots$ |

## Mathematics: Question 3

| Suggested Solutions | Marks Awarded | Marker's Comments |
|---|---|---|
| (a) (i) $AC = \sqrt{8^2+3^2}$<br>$= \sqrt{73}$ units. (1) | 1 | Well done |
| (ii) $AC: \frac{y-2}{x+2} = \frac{5-2}{6+2} = \frac{3}{8}$<br>$3x+6 = 8y-16$<br>$3x-8y+22=0$ (2) | 1<br>1 | 1 mk for gradient<br>1 mk correct method |
| (iii) $d = \left\lvert \frac{3\times 3 - 8\times(-2)+22}{\sqrt{3^2+8^2}} \right\rvert$<br>$= \frac{47}{\sqrt{73}}$ units (2) | 1<br>1 | Some did not know formula. |
| (iv) Area $\triangle ABC = \frac{1}{2} \times \sqrt{73} \times \frac{47}{\sqrt{73}}$<br>$= 23.5$ unit$^2$ (2) | 1<br>1 | full mks if incorrect pt (iii) used. |
| (b) (i) $x+y=5$ (1)<br>$2x-3y=-5$ (2)<br>(1)×2: $2x+2y=10$ (3)<br>(2)−(3): $-5y=-15$<br>$y=3$<br>$\therefore x=2$ from (1)<br>Point of intersection is $(2,3)$ (3) | 1<br>1<br>1 | Generally well done.<br>Some simple errors in calculation. |
| (ii) $x+y-5\le 0$, $2x-3y+5\le 0$<br><br><br>(2) | 1<br>1 | 1 mk for EACH correct region.<br>Quite a few careless ~~errors~~ in shading |
| | | |

## Mathematics: Question 4

| Suggested Solutions | Marks Awarded | Marker's Comments |
| --- | --- | --- |
| | | |

(a) $2^{2x} - 7 \times 2^{x} - 8 = 0$

Let $u = 2^{x}$: $u^2 - 7u - 8 = 0$

$(u-8)(u+1) = 0$

$u = 8$ or $u = -1$

$2^{x} = 8$ or $2^{x} = -1$

$x = 3$ is the only solution. 3

(b) (i) $kx^2 + 4x + k$

$\Delta = 16 - 4k^2$ 1

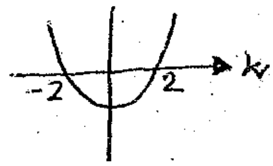
(ii) For no real roots, $\Delta < 0$

$16 - 4k^2 < 0$

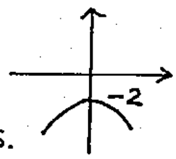
$k^2 > 4$

$k < -2,\ k > 2$ 2

Draw sketch to solve $k^2 > 4$




(c) $x^2 = -8(y+2)$




(i) Vertex $(0, -2)$

(ii) Focal length = 2 units.

(iii) Focus $(0, -4)$

(iv) Directrix: $y = 0$ 4x1

Students who draw sketch made far fewer errors.

$x^2 = -4a(y-k)$

NB length $\Rightarrow +2$

(d) $ax^2 + bx + c = 0$

$$(\alpha-\beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$$
$$= \alpha^2 + 2\alpha\beta + \beta^2 - 4\alpha\beta$$
$$= (\alpha+\beta)^2 - 4\alpha\beta$$
$$= \left(-\frac{b}{a}\right)^2 - 4\left(\frac{c}{a}\right)$$
$$= \frac{b^2}{a^2} - \frac{4c}{a}$$
$$= \frac{b^2 - 4ac}{a^2}$$ 2

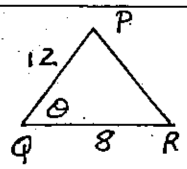
OR

$$\left(\frac{-b+\sqrt{b^2-4ac}}{2a} - \frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$
$$= \left(\frac{\cancel{2}\sqrt{b^2-4ac}}{\cancel{2}a}\right)^2$$

## Mathematics: Question 5

| Suggested Solutions | Marks Awarded | Marker's Comments |
|---|---|---|

**(a)** Using $Area = \frac{1}{2}ab\sin C$




$\frac{1}{2} \times 12 \times 8 \times \sin\theta = 28.8$

$\sin\theta = \frac{28.8}{48}$

$= 0.6$

$\theta = 37°, 143°$ — 3

Marks: ✓ ✓✓

Marker's comments: correct substitution into area formula. (✓) acute (✓) obtuse

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**(b) (i)** $l = r\theta$

$4 = 2.5\,\theta$

$\theta = \frac{4}{2.5}$

$= 1.6$ radians. — 1

Marks: ✓

Marker's comments: correct substitution

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**(ii)** $AB^2 = 2.5^2 + 2.5^2 - 2 \times 2.5 \times 2.5\cos 1.6^c$

$= 12.5 - 12.5\cos 1.6^c$ (COSINE RULE)

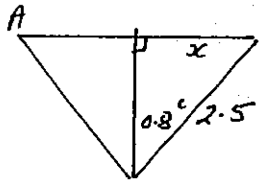
$= 12.865$

$AB = 3.6$ cm (1d.p) — 2

Marks: ✓ ✓

Marker's comments: solution using the cosine rule

OR




$\frac{x}{2.5} = \sin 0.8$ ✓

$x = 2.5\sin 0.8$

$= 1.793$ ✓

$\therefore AB = 3.6$cm (1dp)

Marker's comments: solution making a right angled triangle.

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**(iii)** $Area = \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2} \times 2.5^2(1.6 - \sin 1.6^c)$

$= 1.876\ cm^2$ (3 dp) — 2

Marks: ✓ ✓

Marker's comments: Note: calculator must in radians mode for this question

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**(c) (i)** In $\triangle ABE$, $\triangle ACD$

$\angle ABE = \angle ADC$ (given)

$\angle BAE = \angle DAC$ (same angle)

$\therefore \triangle ABE \,|||\, \triangle ACD$ (two angles equal) — 2

Marks: ✓ ✓

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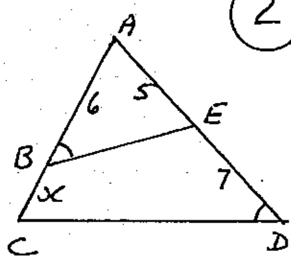
**(ii)** Let $BC = x$ cm




$\frac{6}{5} = \frac{12}{6+x}$ ✓

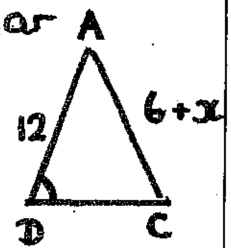
$36 + 6x = 60$

$6x = 24$

$x = 4$

$\therefore BC = 4$ cm ✓ — 2

Marker's comments: equiangular


if you draw the 2 similar triangles next to each other you will get the ratio right.

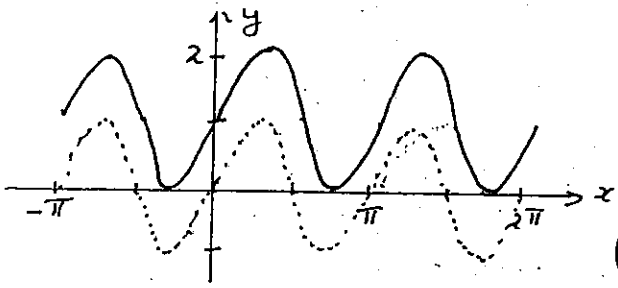
Mathematics: Question 6.

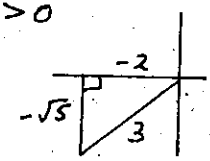
| Suggested Solutions | Marks Awarded | Marker's Comments |
|---|---|---|
| (a) $S_{16} = 288$, $T_6 = 8$ | | |
| Let the series be $a + (a+d) + (a+2d) + \ldots$ | | |
| $a + 5d = 8$ (1) | ✓ | $T_n = a + (n-1)d$ |
| $\frac{16}{2}[2a + 15d] = 288$ | ✓ | $S_n = \frac{n}{2}[2a + (n-1)d]$ |
| i.e. $2a + 15d = 36$ (2) | | |
| (1) x 2: $2a + 10d = 16$ (3) | | |
| (2) - (3): $5d = 20$ | | |
| $d = 4$ | ✓ | |
| From (1) $a = -12$ | | |
| Series is $-12, -8, -4$ — 4 | ✓ | |
| (b) $4 \times (1\cdot2)^n > 560$ | | |
| $(1\cdot2)^n > 140$ | ✓ | |
| $n \log 1\cdot2 > \log 140$ | | |
| $n > \frac{\log 140}{\log 1\cdot2}$ | | |
| $n > 27\cdot103\ldots$ | ✓ | |
| Least value of $n$ is 28 — 3 | ✓ | "least positive integer $> 27\cdot103$ is 28 NOT 27 |
| (c) (i) First deposit is in for 10 years | | |
| $A = 1500(1\cdot04)^{20}$ | ✓ | $A = P(1 + \frac{r}{100})^n$ |
| $= \$3286.68$ — 2 | ✓ | where 8% p.a compounded half yearly is 4% p. half year |
| (ii) Total value | | |
| $= 1500 \times (1\cdot04)^{20} + 1500 \times (1\cdot04)^{18} + \ldots$ | | |
| $\ldots + 1500(1\cdot04)^4 + 1500 \times (1\cdot04)^2$ | ✓ | correct series |
| $= \frac{1500(1\cdot04)^2[(1\cdot04^2)^{10} - 1]}{1\cdot04^2 - 1}$ | ✓ | $S_n = \frac{a[r^n - 1]}{r - 1}$ |
| $= \frac{1500(1\cdot04)^2[1\cdot04^{20} - 1]}{1\cdot04^2 - 1}$ | | |
| $= \$23682.33$ | ✓ | correct answer. |
| using $S_n = \frac{a(r^n - 1)}{r - 1}$ where $r = (1\cdot04)^2$ — 3 | | |

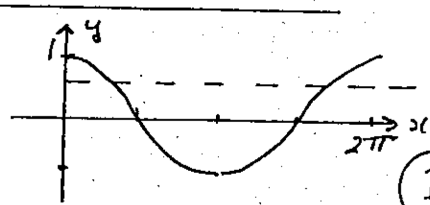
Mathematics: Question 7

| Suggested Solutions | Marks Awarded | Marker's Comments |
| --- | --- | --- |
| (a) $y = 3x^2 - 8x + 2$ <br> $\frac{dy}{dx} = 6x - 8$ | | |
| At $(2,-2)$, $\frac{dy}{dx} = 6\times 2 - 8 = 4$. | 1 | |
| Gradient of normal $= -\frac{1}{4}$. | 1 | |
| Normal: $y + 2 = -\frac{1}{4}(x-2)$ | 1 | |
| $4y + 8 = -x + 2$ <br> $x + 4y + 6 = 0$ <br> (3) | | |
| (b) $y = x^3 - 12x^2 + 48x + 50$ <br> $\frac{dy}{dx} = 3x^2 - 24x + 48$ <br> $= 3(x^2 - 8x + 16)$ <br> $= 3(x-4)^2$ | | It is not true that: $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$ defines a H.P.I (check with $y = x^4$) |
| At stationary point, $\frac{dy}{dx} = 0$ | 1 | |
| $\therefore 3(x-4)^2 = 0 \quad \therefore x = 4$ | 1 | |
| There is only one stationary point <br> $\frac{d^2y}{dx^2} = 6x - 24$. | | OR first derivative test |
| when $x = 4$, $\frac{d^2y}{dx^2} = 6\times 4 - 24 = 0$ | 1 | |
| when $x = 3$, $\frac{d^2y}{dx^2} = 6\times 3 - 24 = -6$ <br> when $x = 5$, $\frac{d^2y}{dx^2} = 6\times 5 - 24 = 6$ | 1 | <table><tr><td>$x$</td><td>3</td><td>4</td><td>5</td></tr><tr><td>$\frac{dy}{dx}$</td><td>+3</td><td>0</td><td>+3</td></tr></table> |
| $\therefore x = 4$ is a point of inflexion since $\frac{d^2y}{dx^2} = 0$ and concavity changes. <br> (4) | 1 | |
| (c) (i) $4x + BH = 40$ <br> $BH = 40 - 4x$ <br> (1) | | |
| (ii) $A = x(40 - 4x)$ <br> $= 40x - 4x^2$ | 1 | |
| $\frac{dA}{dx} = 40 - 8x$ <br> $= 0$ when $x = 5$. | 1 | |
| $\frac{d^2A}{dx^2} = -8 < 0$ | 1 | Must prove Max not just assume |
| $\therefore$ Area is a maximum when <br> $AB = 5\text{ m}$, $BH = 20\text{ m}$. <br> (4) | 1 | Read Q carefully <br> Area not required |

# Mathematics: Question 8

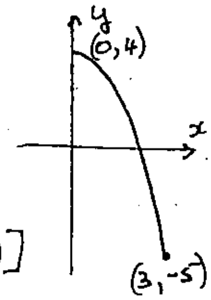
<table>
<tr><th>Suggested Solutions</th><th>Marks Awarded</th><th>Marker's Comments</th></tr>
<tr><td>(8) $y = 1 + \sin 2x$<br>(i) Period $= \frac{2\pi}{2} = \pi$   1</td><td>1</td><td>Well done.</td></tr>
<tr><td>(ii)<br>

3</td><td>3</td><td>1 for shape<br>1 position<br>1 $-\pi \to 2\pi$<br>some quite messy.</td></tr>
<tr><td>(b) $\cos\theta = -\frac{2}{3}$, $\tan\theta > 0$<br>$\theta$ in 3rd quadrant.<br>$\sin\theta = -\frac{\sqrt{5}}{3}$<br>

OR Use $\sin^2\theta + \cos^2\theta = 1$<br>$\sin^2\theta + \frac{4}{9} = 1$<br>$\sin^2\theta = \frac{5}{9}$ $\therefore$ $\sin\theta = \pm\frac{\sqrt{5}}{3}$<br>But $\sin\theta < 0$ $\therefore$ $\sin\theta = -\frac{\sqrt{5}}{3}$   2</td><td>1<br>1</td><td>1 for correct $\sqrt{5}$<br>1 for correct 3<br>not well done</td></tr>
<tr><td>(c)(i) $y = \cos x$<br>

1</td><td>1</td><td>axes NOT labeled loss of mark.</td></tr>
<tr><td>(ii) $\cos x \le \frac{\sqrt{3}}{2}$<br>If $\cos x = \frac{\sqrt{3}}{2}$, $x = \frac{\pi}{6}, \frac{11\pi}{6}$<br>For $\cos x \le \frac{\sqrt{3}}{2}$, $\frac{\pi}{6} \le x \le \frac{11\pi}{6}$   2</td><td>1<br>1</td><td>1 for $x \geq \frac{\pi}{6}$<br>1 for $x \leq \frac{11\pi}{6}$<br>Extra INCORRECT info loss of mk.</td></tr>
<tr><td>(d)(i) $\sin^2\theta + \cos^2\theta = 1$<br>Divide by $\cos^2\theta$   1</td><td>1</td><td>Most OK.</td></tr>
<tr><td>(ii) $\int \tan^2 x\,dx = \int(\sec^2 x - 1)\,dx$<br>$= \tan x - x + c$   2</td><td>1<br>1</td><td>Most have $\tan x$, $-x$ and $+c$ for both marks.</td></tr>
</table>

# Mathematics: Question 9.

| Suggested Solutions | Marks Awarded | Marker's Comments |
|---|---|---|
| (a) $y = 4 - x^2$<br>$V = \pi\int x^2\,dy$<br>$= \pi\int_{-5}^{4}(4-y)\,dy$<br>$= \pi\left[4y - \frac{1}{2}y^2\right]_{-5}^{4}$<br>$= \pi\left[(16-8) - (-20-12\frac{1}{2})\right]$<br>Volume $= 40\cdot5\pi$ unit$^3$.<br><br>3 | 1<br>1<br>1 | - some rotated about the x axis.<br>- generally OK. |
| (b) $y = 2x$, $y = x(x-3)$<br>(i) $x^2 - 3x = 2x$<br>$x^2 - 5x = 0$<br>$x(x-5) = 0$<br>$x = 0$ or $x = 5$.<br>$\therefore$ x-coord of $A$ is $x = 5$<br>1 | 1 | Well done. |
| (ii) $A = \int_0^5 (y_1 - y_2)\,dx$<br>$= \int_0^5 \left[2x - (x^2-3x)\right]dx$<br>$= \int_0^5 (5x - x^2)\,dx$<br>$= \left[\frac{5}{2}x^2 - \frac{1}{3}x^3\right]_0^5$<br>$= \left(\frac{125}{2} - \frac{125}{3}\right) - (0-0)$<br>Area $= \frac{125}{6}$ unit$^2$<br>3 | 1<br>1<br>1 | - many tried other ways with some success.<br>- Some divided region into 3 areas. NOT SUCCESSFUL. |
| (c)(i) $\int_0^8 f(x)\,dx \doteqdot \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$<br>$\doteqdot \frac{2}{3}\left[3\cdot2 + 4\times1\cdot4 + 2\times0\cdot6 + 4\times2\cdot2 + 3\cdot6\right]$<br>$\doteqdot 14\cdot93$<br>2 | 1<br>1 | - most did this with little difficulty |
| (ii) $V = \pi\int y^2\,dx$<br>$\doteqdot \pi\times\frac{2}{3}\left[3\cdot2^2 + 4\times1\cdot4^2 + 2\times0\cdot6^2 + 4\times2\cdot2^2 + 3\cdot6^2\right]$<br>$\doteqdot \frac{2\pi}{3}\times51\cdot12$<br>Volume $\doteqdot 34\cdot08\pi$ or $107$ unit$^3$<br>3 | 1 1<br>1 | - Many could not understand how to get a volume with this rule.<br>- poorly done. |

Mathematics: Question 10 .

| Suggested Solutions | Marks Awarded | Marker's Comments |
|---|---|---|
| | | |

**Suggested Solutions**

(a) $N = 2000\, e^{0.005t}$

(i) When $t=10$, $N = 2000 \times e^{0.05}$
$= 2103$ 1

(ii) $\frac{dN}{dt} = 2000 \times e^{0.005t} \times 0.005$
$= 10\, e^{0.005t}$ 1

When $t=10$, $\frac{dN}{dt} = 10 \times e^{0.05}$
$= 10.5$ 1 2

Colony is increasing at 10.5/minute.

(iii) When $N = 4000$, $4000 = 2000\, e^{0.005t}$
$e^{0.005t} = 2$
$0.005t = \ln 2$
$t = \frac{\ln 2}{0.005}$
$= 138.6$ 1

It takes 138.6 minutes to double.

(b) $x = 2t - \log_e(4t+1)$

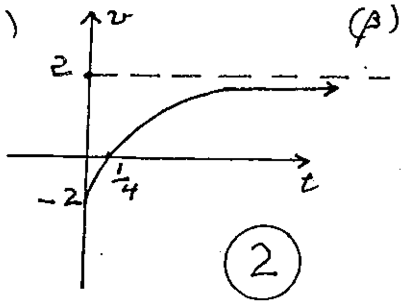
(i) $v = 2 - \frac{4}{4t+1}$
$= \frac{8t+2-4}{4t+1}$
$= \frac{8t-2}{4t+1}$ 1

$a = \frac{(4t+1)\times 8 - (8t-2)\times 4}{(4t+1)^2}$
$= \frac{32t+8-32t+8}{(4t+1)^2}$
$= \frac{16}{(4t+1)^2}$ 1 2

(ii) When $t=0$, $v = -2\,m/s$.
$a = 16\ m/s^2$ 2

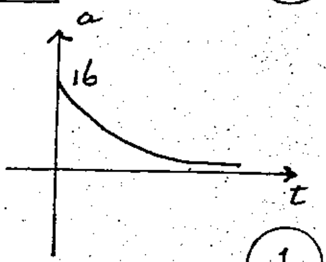
(iii) When $v=0$, $t = \frac{1}{4}$ 1

(iv) ($\alpha$)




2

($\beta$)




1

**Marker's Comments**

Part (a) was well done.

b i) Need to show working for getting from $2 - \frac{4}{4t+1}$ to the required result $\frac{8t-2}{4t+1}$. Those that didn't, got no marks.

ii) well done

iii) when solving $\frac{8t-2}{4t+1} = 0$ many made the error of having as the next line $8t-2 = 4t+1$ $(0\times(4t+1) = 0)$

iv) no marks awarded if linear graphs were drawn for velocity and acceleration. correct shape and asymptote, $v = 2$ needed to get full marks for velocity graph.